

Abstract Algebra I

Exercise Sheet 2: Cycles and Permutations

Question 1. First write the following permutations of S_{10} in cycle notation. Then write the permutations as products of transpositions. Determine the parity of the permutations. In each case also find the inverse permutation and their 100-th power. For all 90 combinations of pairs of permutations find their product.

- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 2 & 6 & 9 & 1 & 5 & 7 & 8 & 10 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 3 & 4 & 1 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 7 & 1 & 3 & 6 & 5 & 4 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 2 & 3 & 5 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & 4 & 6 & 1 & 2 & 9 & 8 & 5 & 7 & 3 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 7 & 5 & 3 & 1 & 8 & 2 & 6 & 9 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$

Question 2. Find permutations σ and τ of S_{10} that commute. Find permutations that do not commute.

Question 3. Recall that S_4 is the group of symmetry preserving transformations of a regular tetrahedron. What physical transformations do 3-cycles of S_4 correspond to?

Question 4. Let $\sigma = (1, 4, 3, 6, 5, 7, 2) \in S_{10}$ and $\tau = (3, 5, 4, 1) \in S_{10}$. Find $\sigma\tau\sigma^{-1}$. Compare the result with $(\sigma(3), \sigma(5), \sigma(4), \sigma(1))$.

Question 5. Repeat the previous question with arbitrary cycles in S_n , with $n \geq 2$.